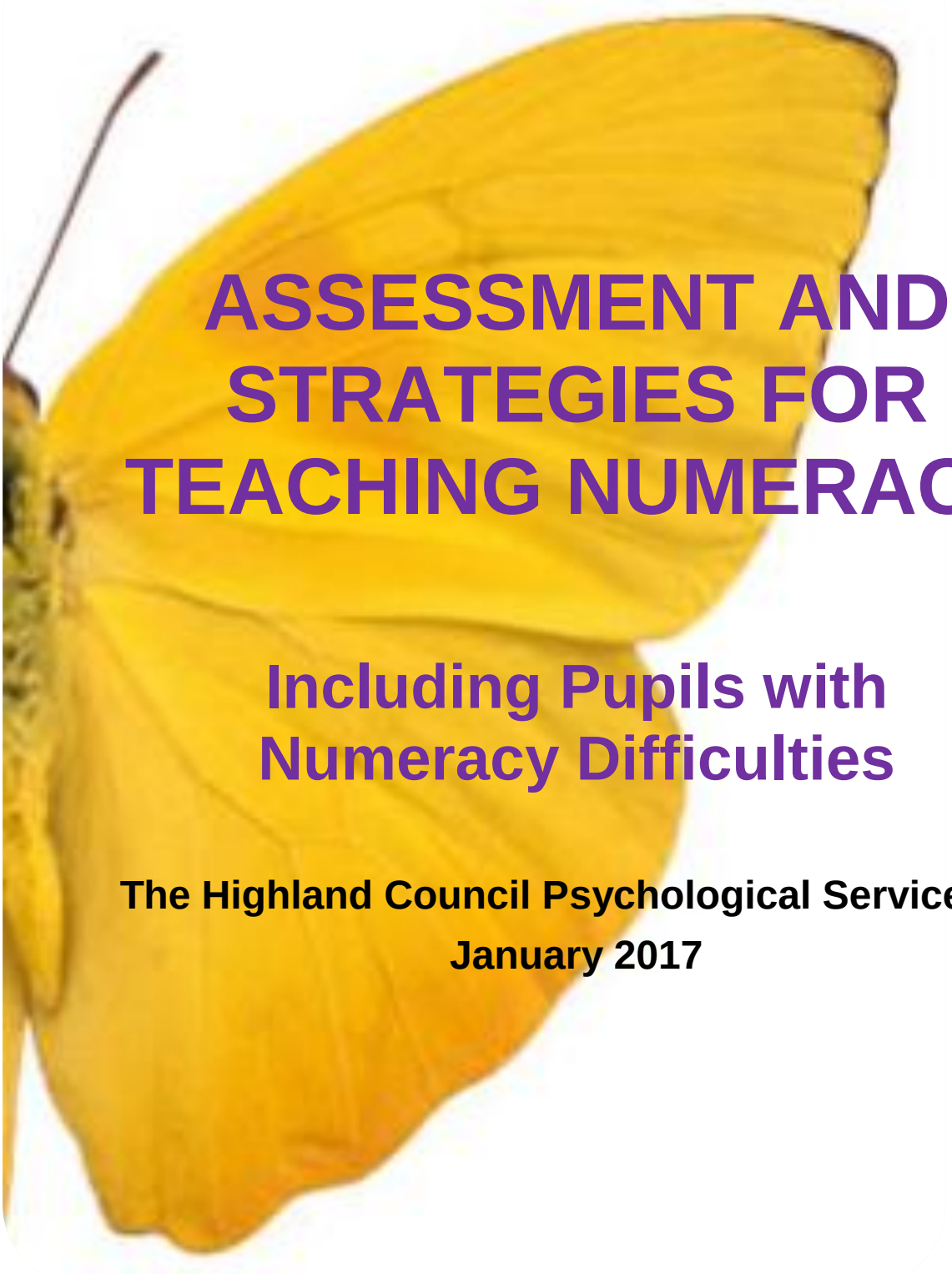
Highland Council Psychological Service

Works for Positive Change



ASSESSMENT AND STRATEGIES FOR TEACHING NUMERACY

Including Pupils with Numeracy Difficulties

The Highland Council Psychological Service

January 2017

CONTENTS

INTRODUCTION

The purpose of this practice paper is to outline The Highland Council Psychological Service (THCPS) approach to meeting the needs of children and young people learning numeracy, including those with persistent difficulties with numeracy. The aim is to address an expressed need from schools for more guidance regarding identification and intervention for pupils experiencing persistent difficulties with numeracy.

In this practice paper, the focus is mainly on difficulties with numeracy: basic number knowledge and arithmetical procedures. Wider mathematical concepts (e.g. shape, geometry, algebra etc.) are referred to but not discussed at length. Difficulties with numeracy can be attributed to a variety of causes (see Appendix D for further discussion about Additional Support Needs) and may not be due to a specific learning difficulty. For these reasons, the authors have used the description 'Persistent Difficulties with Numeracy', as discussed further in the definition section.

National Context

Raising the standards of educational attainment for all, in the core skills of literacy and numeracy, is a National Priority (The Education (National Priorities) (Scotland) Order 2000). This was emphasised in the Raising Attainment for All programme that was launched in 2014 by the Scottish Government. Curriculum for Excellence (SEED, 2009) stated that numeracy and literacy deserved equal time and weighting of importance. Numeracy Hubs (introduced in 2014) have raised the profile of maths teaching and professional development.

Local Context

The aim of the Highland Numeracy Strategy is "to ensure that all young people develop the numeracy skills they need to achieve success in life, learning and work" (Highland Council, 2016, p.1,). The authority aims to increase shared understanding amongst staff about development of numeracy to "support practitioners in planning and integrating learning, teaching and assessment" through Professional Learning, The Highland Numeracy Blog and The Highland Numeracy Progression (Highland Council, 2016, p.1,). THCPS is part of this strategy and is represented on the Numeracy Steering Group. We have been tasked to focus on assessment and intervention for learners, including those with persistent difficulties with numeracy.

The Highland Numeracy Progression is based on the Maths Recovery programme and New Zealand Maths. Maths Recovery is an evidence-based intensive intervention, underpinned by the Stages of Early Arithmetical Learning (Wright, Martland & Stafford, 2006). New Zealand Maths is a universal programme, used in some Highland Schools. The content and strategies from these programmes was mapped onto Curriculum for Excellence Levels, appropriate Experiences and Outcomes, and Significant Aspects of Learning making it suitable for a Scottish Context. It is designed to be a guide for planning, a reference document and a key resource for teaching numeracy and maths (Highland Council, 2016).

The assessment guidance in this practice paper is based upon The Highland Numeracy Progression and the associated Diagnostic Assessments. These resources are available from The Highland Numeracy Blog alongside training for their use.

DEFINITIONS AND IDENTIFICATION

The Nature of Numeracy and Maths

Learners develop competence and confidence in numeracy by learning facts about numbers (e.g. how quantities are represented in different ways etc.), how numbers represent quantity in the real world and how numbers interact through learning procedures (e.g. adding numbers together etc.). The knowledge and skills then have to be applied in a logical and reasoned way in order to make sense of the world and once that is achieved then an individual is numerate (Emerson & Babtie, 2013).

In Curriculum for Excellence, learners are considered numerate if they have developed *"*the confidence and competence in using number which will allow individuals to solve problems, analyse information and make informed decisions based on calculations.*"* (Scottish Government, 2009, p.1,). The essential knowledge and skills required to become numerate are: accurate counting; remembering key number facts; logical reasoning and problem solving ability (Emerson & Babtie, 2013).

The foundation of numeracy is counting which is a set of skills and knowledge that takes years to master fully in normal development. All the principles listed below must be applied to count fluently (see Figure 1):

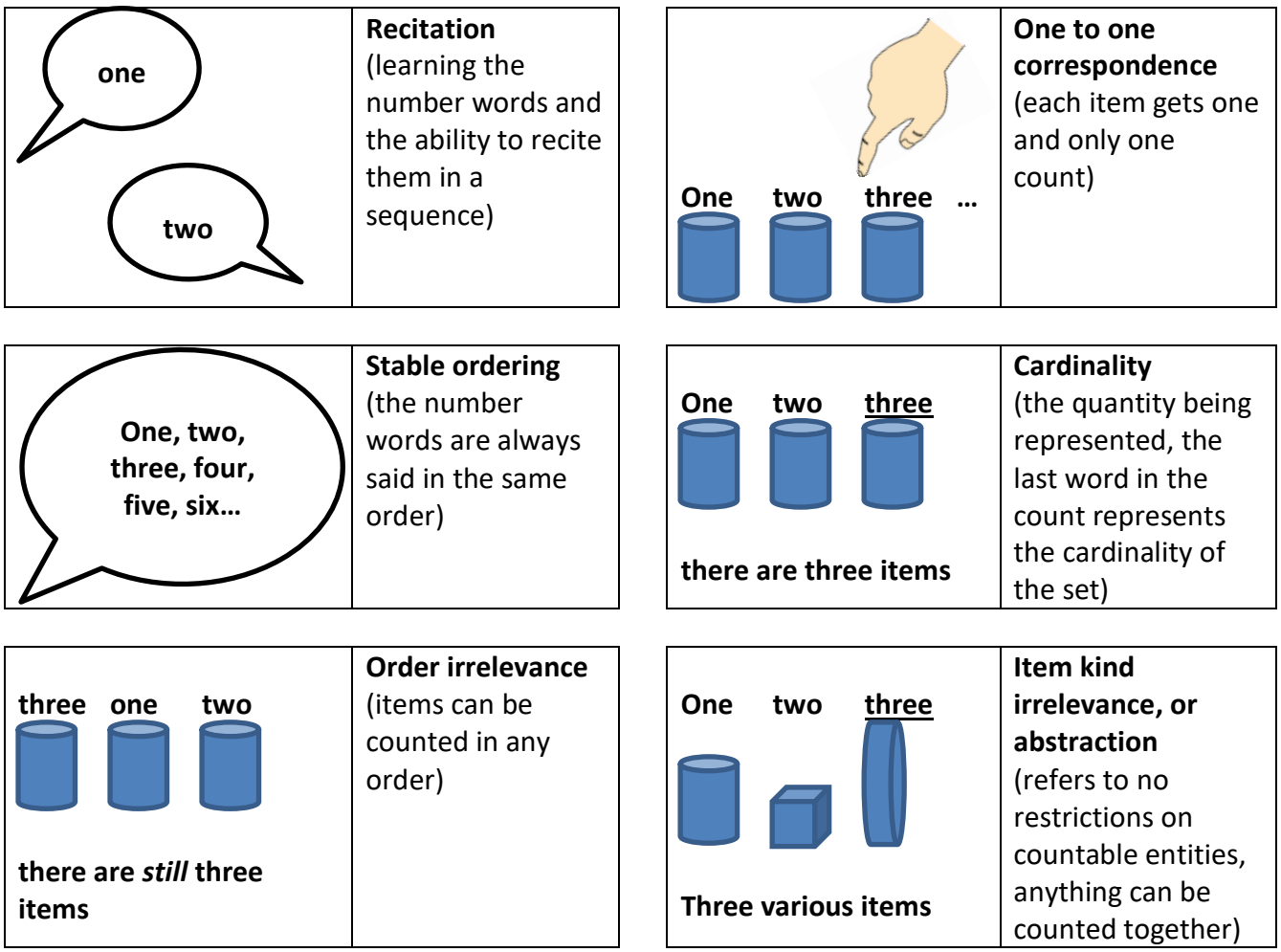


Figure 1: Counting principles diagrams (Gelman & Gallistel, cited in Cordes & Gelman, 2005; Emerson & Babtie, 2013; Sullivan et al., 2005)

Numeracy activities, even in the earliest stages, involve a wide variety of facts, the language used, abstract concepts (numbers as internal representations of quantity) and complex mental activities (comparisons and calculation procedures). It can take a long time to become secure with the knowledge and skills required, because they need to be reinforced and understood. The knowledge and skills are progressive, building on each other. Children tend to learn to count and sequence numbers before learning to add and subtract, then move on to learning multiplication, division and so on (for example). If learners experience difficulty acquiring the required knowledge or skills due to anxiety, behavioural difficulties, inappropriate teaching, disrupted education or learning difficulties then they could struggle to progress and may perform poorly (Butterworth, 2005).

Learning difficulties specifically refer to difficulties with cognitive skills that affect learning including difficulties with attention, concentration, memory (short-term, long-term and/or working memory), reasoning and sequencing (APA, 2013; Emerson & Babtie, 2013).

A Specific Difficulty in Maths (known previously as Dyscalculia)

There are various reasons why a person may experience difficulties with numeracy and perform poorly in mathematical tasks and only a small proportion will be due to a specific learning difficulty in maths, sometimes referred to as dyscalculia, (e.g. Butterworth, 2005; Emerson & Babtie, 2013).

At the time of writing (January 2017), there was no longer any reference to the term 'dyscalculia' on the Education Scotland website, or on the Department for Education website (BDA, 2015) which had a widely published definition from 2001, (e.g. Butterworth, 2005; Butterworth & Yeo, 2004; Emerson & Babtie, 2013). In the DSM-V, specific learning difficulties in reading, writing and maths are now grouped under one umbrella term: Specific Learning Disorder (APA, 2013).

As there does not seem to be a satisfactory definition or criteria, this practice paper refers to 'persistent difficulties with numeracy' as a descriptive term and follows a needs-led model (see Appendix C and Appendix D for Additional Support Needs information), similar to the Scottish Qualifications Authority (SQA) (see Appendix E).

FEATURES OF PERSISTENT DIFFICULTIES WITH NUMERACY

A learner experiencing difficulties in numeracy may have some, or all, of the features described below and may or may not have other Additional Support Needs. **Not all children who present with numeracy challenges should be considered as having a specific learning difficulty with numeracy and/or maths.** These learning needs accompany the Additional Support Needs listed and described in Appendix C, which also includes strategies and resources.

Number Sense

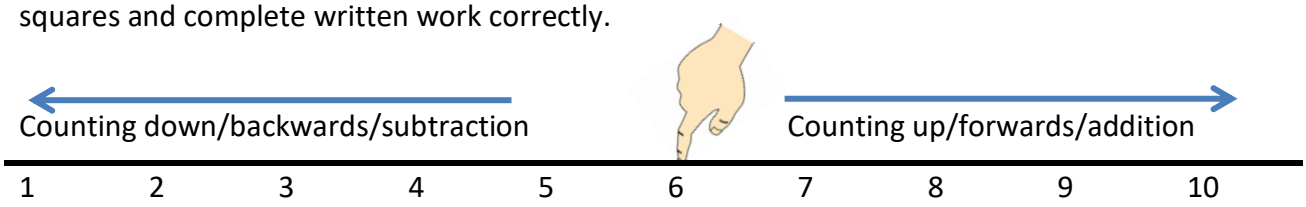
Number sense is an understanding of basic numerical concepts and an intuitive 'feel' for numbers. It is the basic understanding that numbers represent quantities and also how those quantities relate to each other (Dehaene, 2011). Pupils without a strong number sense do not estimate quantities appropriately and cannot subitise (immediately know the quantities up to three or four at a glance) (Geary & Hoard, 2005). This lack of understanding means that they can lack confidence using basic mathematical procedures and may rely on counting methods (e.g. count on their fingers) and other immature strategies (e.g. counting from one) when doing mathematical calculations (Geary & Hoard, 2005).

Remembering facts about numeracy and maths

Just as some children have difficulty retaining sight words in reading, some have difficulty retaining and recalling number facts and procedures (Geary & Hoard, 2005). This means that learners may forget basic number facts or work out every step of a calculation using counting methods or other slow or inappropriate methods, such as guessing. Difficulties with working memory (the ability to hold information in the mind and manipulate it) may also have an impact on mathematical calculation, in mental arithmetic and multi-step problems, and would require support (Emerson & Babtie, 2012; Geary & Hoard, 2005).

Visuo-Spatial

Spatial processing and numerical processing are intimately connected and visuo-spatial reasoning ability and mathematical ability have been found to have a strong association in several studies (Dehaene, 2011; Fias & Fischer, 2012). Most people have a (horizontal) mental number line which they use in basic calculation and, in western society (where we read left to right), there is a general tendency to count up/forwards to the right and count down/backwards to the left with a left-hand bias for smaller numbers and a right hand bias for larger numbers (Dehaene, 2011; Fias & Fischer, 2012). This automatic association of space and numbers seems to help understanding of how numbers relate to each other. When a person has difficulty with spatial awareness, then they may not show the same biases for left and right and may get mixed up or struggle to count, use number lines, use number squares and complete written work correctly.



Maths Anxiety

Maths anxiety is defined as a negative emotional response to situations involving number and mathematics (Ashcraft & Ridley, 2005; Young, Wu & Menon, 2012). It can encompass a range of emotional states that vary in intensity from mild apprehension or dislike to acute feelings of genuine fear or dread (Ashcraft & Ridley, 2005). Anxiety is often associated with negative effects on cognitive performance, particularly tasks that place greater demands on cognitive resources (take up more mental space and mental effort) (Eysenck, Derakshan, Santos, & Calvo, 2007). Highly anxious individuals may answer maths questions rapidly with more errors (the highly anxious children could have been guessing or sacrificing accuracy for speed with the more complex problems), or much more slowly than their low anxiety peers (suggesting a reliance on less efficient counting methods) (Ashcraft & Ridley, 2005).

LEARNING AND TEACHING

The assessment and intervention process carried out to identify and support any additional support needs should follow the Highland Practice Model Stages of Intervention (see Appendix B) (Highland Council, 2016).

Gathering Assessment Information

Classroom Assessment and Monitoring (Stage 1)

Assessment is part of learning and should link with Curriculum for Excellence, especially in relation to the five significant aspects of learning in numeracy and mathematics:

- Using knowledge and understanding of the number system, patterns and relationships
- Using knowledge and understanding of measurement and its application
- Using knowledge and understanding of shape and space
- Researching and evaluating data to assess risks and make informed choices
- Applying numeracy and mathematical skills.

Numeracy tracking documents for each of the above are available from the Highland Numeracy Blog (see Appendix F). Use of tracking sheets to monitor progress and identify those who may require extra input and support are recommended and the Highland Numeracy blog has some excellent templates. The significant aspects of learning are discussed on the Education Scotland website; in an Education Scotland Practice Paper and also on the Highland Numeracy Blog. These may prove a useful starting point for a Class Teacher to complete if there are concerns regarding a child or young person's numeracy. This should be considered alongside the pupil's class work including Assessment for Excellence (InCAS results, pupil's self and peer assessment). Information from the pupil's home should also be gathered.

Assessment for Excellence assessments by the Centre for Evaluation and Monitoring (CEM) at University of Durham (e.g. InCAS) can also provide some information. It is important to remember that these assessments are intended for screening purposes only and should not be used diagnostically. They can identify any areas of difficulty that may require further investigation. No test should be considered in isolation, all assessment results should be considered alongside class work and other contextual information.

Contextual Assessment Information (Stage 1, possibly Stage 2)

If a learner is identified as having difficulties with numeracy through classroom assessment and monitoring, it is very important to gather contextual information regarding the individual learner and the learning environment to investigate the difficulties and start to address them. Gathering this information effectively may prevent unnecessary individual assessment.

Contextual information about the individual learner:

- Information obtained by discussion with child & parents/carer:
    - Personal circumstances, e.g. English as an Additional Language etc.
    - Educational history, including any disruption
    - Any existing Additional Support Needs, any previous Additional Support (Support for Learning) and/or involvement of other agencies e.g. Educational Psychology, Speech and Language Therapy, Occupational Therapy etc.
    - General feelings about school, including confidence and any anxieties
    - General feelings about maths, including confidence and any anxieties
    - Parents' feelings about school and maths, including any anxieties
    - Support provided at home and home/school collaboration
    - The learner's knowledge of their own learning strategies (metacognition)

- Information from the class teacher
    - Teacher observations and feedback
    - Examples of maths work, including mental maths and written work

Contextual information about the learning environment

- Classroom observations by school staff
- Layout of the classroom and organisation of numeracy teaching
- Access to programmes and concrete materials appropriate for the child, including differentiation
- Preferred learning and teaching styles of the teacher and the learner and how the learner responds to the teaching strategies and support
- Evidence of the impact of interventions tried previously, linked to the assessment results, within an identified timeframe

The report template in the Appendix A includes contextual information questions that cover some of these areas. Relevant contextual information should also be included in a Form 1 or Child's Plan (Highland Council, 2015).

Individual Assessment to Identify Barriers to Learning (Stage 2)

Individual assessment could be carried out by the class teacher or Additional Support Needs Teacher. Summative methods of assessment such as standardised testing of numeracy and maths can only provide part of the picture. The wide variety of skills (e.g. counting, estimating, recalling number bonds etc.), procedures (e.g. carrying and borrowing in subtraction), and differences between school systems in teaching these procedures means that there can be wide differences between standardised tests for basic arithmetic (Butterworth, 2005). It is far more important to observe the learner and use a diagnostic interview method more in keeping with dynamic individual teaching (scaffolding) to create a profile of a learner's relative strengths and areas of difficulty. By observing and discussing which strategies the learner is using to try and complete numeracy tasks, it may be possible to find out the cognitive or conceptual difficulties that could lead to poor performance in numeracy tasks.

With a diagnostic interview method, the emphasis is on observing how the child carries out each maths problem and the strategies they use. During the assessment, the assessor should make descriptive notes detailing the child's behaviour and ask questions about how the child worked out their answers. The assessor may even want to video the assessment process as this will record subtle behaviour from the learner such as counting fingers under the table, eyes moving while silently counting etc. (If video is used, be sure to ask for appropriate permissions from parents/carers and protect data in accordance with Data Protection legislation.)

It is necessary to identify and create a profile of the learner's strengths and pressures including levels of attainment. In Highland, classroom level assessment can be supplemented with the Highland Numeracy Progression Diagnostic Assessment sheets available on the Highland Numeracy Blog. These assessments have been designed to work alongside the Highland Numeracy Progression and are linked to Curriculum for Excellence levels. Other assessment tools that could be used instead are: the New Zealand Maths assessments; Maths Recovery assessments; and The Dyscalculia Assessment book (Emerson and Babtie, 2012). See the annotated bibliography in Appendix F for more details. Most of the assessment tools listed have a diagnostic interview format and should include detailed observation notes about the pupil's strategies and reactions to numeracy tasks. The report template appended to this policy has been written for use with the Highland Numeracy Progression diagnostic assessment

sheets (see Appendix A). Any notable assessment findings should be recorded within the Child's Plan, following the Highland Practice Model (2014) for Additional Support Needs (see Appendix B for the Staged Approach).

Intervention

Once all the assessment information has been gathered and difficulties with numeracy have been identified it is important to use that information to put in appropriate and targeted interventions to address those difficulties. This information would form the actions as part of a Form 1, Individualised Education Plan (IEP) and/or Child's Plan (Highland Council, 2015). See Appendix C for some suggested strategies and resources for addressing different difficulties.

Learning Environment (Universal Intervention, Stage 1)

When children are developing mathematical thinking and numeracy, they construct their own understanding of number and arithmetic procedures. The learning environment needs to be emotionally safe so the learners feel safe to make mistakes and experiment. Mistakes are good because they are opportunities for learning (Boaler, 2016). Anxieties need to be discussed and recognised (in teachers, parents and pupils) and addressed so they are not passed on to others. Different methods of finding the correct answer and learning styles should also be appreciated (Chinn, 2011).

The classroom could have maths language and vocabulary displayed for reference (although if displays are not interacted with by teacher and pupils they will not be valued) and concrete supports available for all pupils to use. These concrete supports could include: whiteboards and pens; number lines, number squares, multiplication squares, counters or cubes, abacus, Rekenrek etc. See the Highland Numeracy Blog for more ideas.

Teaching (Universal and Targeted Intervention, Stage 1 and 2)

Classroom Teaching (Stage 1)

Classroom learning should be appropriately differentiated. Learners should consolidate and learn new applications for familiar skills and knowledge and be slightly stretched when acquiring new skills and knowledge. If a learner becomes panicked or frustrated too often in numeracy and maths lessons, this can develop into maths anxiety and lead to task avoidance (Lee & Johnston-Wilder, 2016). Regular monitoring will help suitable progression and identify where further consolidation is needed. Teaching should always build on a learner's existing knowledge and fill in gaps of areas they do not understand. Where possible, numeracy and maths tasks should be applied to real world situations in order to encourage the view that numeracy is valuable and important to keep pupils motivated.

Research has found that pupils with a growth mindset are more mathematically resilient (e.g. Kooken, Welsh, Mccoach, Johnston-Wilder & Lee, 2013; Lee & Johnston-Wilder, 2016). Pupils with fixed mind-sets were more likely to give up on tasks, whereas those with a growth mind-set were more likely to persevere. This has implications for the type of praise teachers give. Dweck (2006) showed that a growth mind-set could be developed with praise for effort and perseverance.

Targeted Teaching and Intervention (Stage 2)

In addition to the guidance for classroom teaching described above, a learner may also require more targeted intervention either as on individual basis or in a small group. If individual assessment has been carried out then this would be the basis of an IEP (recorded in the Child's Plan) (Highland Council, 2015). The learning targets should be specific and include how the knowledge or skill will be taught as well as a review date. The assessment should indicate the numeracy knowledge and/or skills that require more targeted teaching and the learner's typical strategies when working out maths problems.

Start with concrete representations of numbers and quantities so learners can see how the calculation works and how the numbers relate to each other. Gradually introduce visual representations (initially alongside concrete materials) such as dot patterns (e.g. like dominos, dice or Numicon), dot arrays (e.g. five and ten frames), Cuisinaire rods, number lines, triad diagrams to show how commutativity works (e.g. 2+3=3+2) etc. See the suggested support materials in Appendix F for teaching materials. As with classroom teaching, there needs to be a balance of consolidation of familiar knowledge and skills and teaching of new knowledge and skills that slightly stretch the learner. Once the learner is secure with concepts using concrete and visual materials, move on to abstracts representations (e.g. written work).

Reassessment after a significant period of targeted intervention (12 months is a suggestion) in addition to regular monitoring is essential to keep track of progress.

Challenge (Universal and Targeted Intervention, Stage 1 and 2)

Pupils will show more mathematical resilience and perseverance if they understand that numeracy and maths require struggle, curiosity and persistence in order to develop and learn something new (Kooken, et al., 2013; Lee & Johnston-Wilder, 2016). Pupils who understand that struggle is part of numeracy believe that the reason the numeracy task is challenging is due to the task being difficult and not because of limitations in their ability. Meaningful praise for progress made due to effort and recognition of successes despite adversity will encourage perseverance and a growth mind-set. By getting a problem wrong, children learn there are other ways of to tackle successfully and this encourages flexibility of thinking and utilises aspects of meta-cognition (Boaler, 2016).

As well as new skills and knowledge, strategies can also be applied to help learners make connections in their learning and think more deeply about numeracy and maths concepts. Hattie (2011) discussed how teaching metacognition skills for numeracy can be useful, especially if pupils have free choice of which strategy to use to solve a problem (instead of an over-reliance on teacher direction). Cognitive Activation was highlighted in the information gathered by the OECD Programme for International Student Assessment (PISA). It was a teaching practice used in maths lessons in England associated with higher maths achievement, increased self-efficacy in pupils with low socio-economic status and lower levels of maths anxiety (NFER, 2015).

Cognitive Activation practices encourage pupils to:

- reflect on problems
- tackle maths problems that require them to think for an extended time
- use their own procedures for solving complex problems
- try problems with no immediately obvious method of solution or multiple solutions
- learn from mistakes within a supportive and safe learning community
- try problems in different contexts and apply what they have learned to new contexts
- explain how they solved a problem and why they chose a particular method

Evaluating Progress and Next Steps

Evaluation for Improvement (Stage 1, 2 and 3)

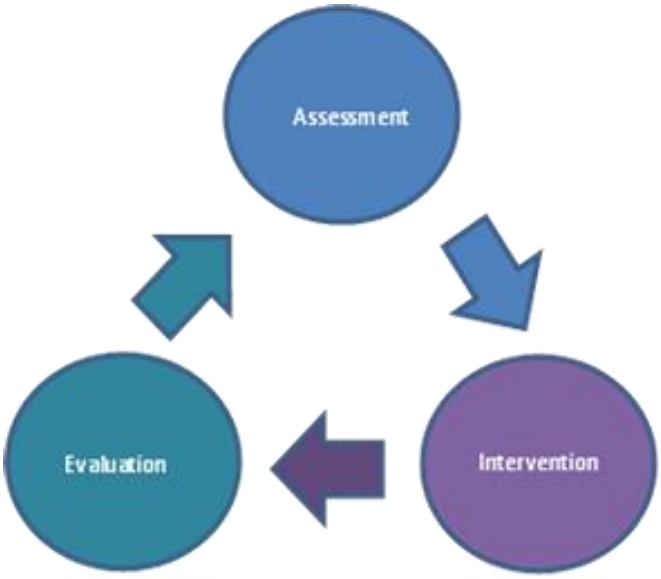


Figure 2: Diagram showing how assessment, intervention and evaluation are linked and cyclical

Educational progress is best monitored over time to ensure that progress is being made using formative and summative assessment methods. Information gathering includes observations of strategies and contextual information then comparing results from before and after intervention.

Some learners make excellent progress after targeted intervention in areas they previously found difficult once a misconception, missing skill or knowledge gap is addressed. It is important to involve the pupil, and the parents/carers, in appropriate discussion about the assessment, intervention, preferred learning styles and seek their views about proposed plans of action.

Re-assessment and monitoring can be more focused and selective as it will be informed by the previous assessment and it should link with targets from the child's Form 1, IEP and/or Child's Plan . This re-assessment would then inform next steps for intervention, as part of ongoing assessment; intervention and evaluation process (see Figure 2).

Role of Specialist Professionals in Learning and Teaching (Stage 3 and 4)

Role of an Educational Psychologist in Learning and Teaching

Educational Psychologists (EPs) can provide support to schools through consultation, assessment, intervention, training and research. Consultation with school staff may occur when children are experiencing difficulty with learning and not making expected progress. EPs will work with school staff and other professionals and not necessarily directly with the child. The aim is to achieve a better understanding of the factors that may be helping or hindering progress and help school staff identify possible ways forward.

The Psychological Service can;

- Contribute to a Solution-Focussed discussion involving the three areas of observation and assessment (Concepts and Skills, Learning Environment, Additional Support) to inform a Child's Plan (Highland Council, 2015).
- Support the skills development of those undertaking ongoing assessment and intervention through consultation and training as appropriate
- Provide information on evidence-based approaches to numeracy teaching and intervention
- Provide information in relation to emotional and behavioural difficulties which may arise as a consequence of persistent difficulties with numeracy

Occasionally EPs may contribute more directly to assessment when the individual pupil has a more complex profile of Additional Support Needs. This would be discussed with the other partners to the plan. Psychologists undertaking detailed assessment will use the most recent professional guidelines and research available to them and developments can be shared through Continuing Professional Development and action research projects. EPs are most likely to examine the interaction between the cognitive processes of the learner and the learning opportunities and teaching methods which have been used.

Role of Other Allied Professionals in Learning and Teaching

Depending on the nature of the pupil's difficulties, getting advice from other professionals may be appropriate. For example, if a pupil has a noticeable difficulty with language then involving a Speech and Language Therapist or English as an Additional Language Teacher may be helpful. Another example would be if a pupil has considerable difficulty with spatial awareness and coordination then seeking advice from an Occupational Therapist could be beneficial.

For early intervention, it may be worth seeking informal advice at an earlier stage to address and support any identified areas of difficulty relating to numeracy.

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Appendix A: Template (proforma) for reporting persistent difficulties with numeracy assessment

Guidance in Italics, remove in final report

Support for Learning Assessment Report		
SCHOOL:	**DATE:**	
NAME:	**D.O.B.**	**CLASS:**

Strengths and Skills

Reason for Concern

Overall Progress in the Curriculum

From class work, tracking and monitoring, and teacher observations. This could include diagnostic interviews, possibly carried out annually.

Contextual Information

Hearing, eyesight, other diagnosis, schooling, teacher changes, interrupted learning, other supports offered, previous Form 1s, Child Plans etc.? - See Highland Council Psychological Service's 'Assessment And Strategies For Teaching Numeracy' Practice Paper for more detail about contextual assessment (p. 7-8)

Parents/Carers' View of Situation and Background Information

How do the parents/carers feel about maths? Include information from pupil's past schooling and development.

Pupil's View of Situation

How does the pupil feel about maths and school in general?

Portfolio of Evidence In Respect To Maths:

Information from classroom monitoring for the Significant Aspects of Learning:

- *Using knowledge and understanding of the number system, patterns and relationships*
- *Using knowledge and understanding of measurement and its application*
- *Using knowledge and understanding of shape and space*
- *Researching and evaluating data to assess risks and make informed choices*
- *Applying numeracy and mathematical skills.*

Other Observations During Assessment

Speak to the pupil to give them the opportunity to explain their answers and strategies.

Below are suggestions of areas you may wish to think about during assessment. If you notice any relevant details from the points below this would provide rich contextual assessment information. This list is not exhaustive, but is to provide a starting point.

- *What areas did the pupil do well at during assessment? What were the pupil's strengths in regards to numeracy?*
- *Did the pupil seem anxious? Could they have maths anxiety?*
- *Did they seem over-confident despite making mistakes? Speed – did the pupil seem to solve problems too quickly (i.e. with errors!) or did they take a very long time?*
- *Did the pupil avoid certain problems? (e.g. did not attempt any division problems?).*
- *Did the pupil give up? Lose focus etc.? (i.e. do they seem to have a fixed or growth mindset?)*
- *Did the pupil understand the instructions? Could they explain to someone else what problem needed solving? Can they show someone using concrete materials how to solve problems? (thinking style and metacognition?)*
- *If a procedure was modelled to the pupil, and they successfully used this procedure could they then apply this strategy to work out a similar problem in a different situation?*
- *Can the pupil explain the methods they used?*
- *Can the pupil use different methods or strategies to solve problems, or are they reliant on simplistic methods such as counting from 1?*

Assessments Carried Out

Following the use of the Highland planning and tracking progression sheets, further evidence regarding X's maths ability was sought from a variety of assessment opportunities including:

Highland numeracy blog assessment materials

The Highland numeracy assessments can help identify areas of strengths and difficulties, through looking at two aspects namely: Strategy assessments (addition and subtraction; multiplication and division; and fractions) and Knowledge assessments (backwards and forwards number sequencing; grouping and place values, numeral identification; and basic facts).

The Highland Numeracy blog gives good guidance regarding the Knowledge Assessments.

Other possible assessments are:

- 'The Dyscalculia Assessment' (Second Edition); Emerson & Babtie
- Maths Recovery Assessments (1.1, 1.2, 2.1, 2.2, 3.1, 3.2)
- New Zealand Numeracy Project Diagnostic Interview

Knowledge Assessments

The ability to count is the foundation of numeracy (as discussed in practice paper). It encompasses:

- Recitation
- One-to-one correspondence
- Stable ordering
- Cardinality
- Order irrelevance
- Item kind irrelevance or abstraction.

Can the pupil demonstrate all of these counting principles?

Tick and date the stage of pupils demonstrated ability during the diagnostic interview assessment.

Forward Number Word Sequence (FNWS)

Forward Number Word Sequence						
Stage 0 – Emergent FNWS Early*	Stage 1 – FNWS up to 10 Early *	Stage 2 – FNWS up to 10 and number words after Early*/**	Stage 3 – FNWS up to 20 and number words after Early **/***	Stage 4 – FNWS up to 100 and number words after First *	Stage 5 – FNWS up to 1000 and number words after First **/***	Stage 6 – FNWS up to 1 000 000 and number words after Second */**/***

Backward Number Word Sequence (BNWS)

Backward Number Word Sequence						
Stage 0 – Emergent BNWS Early*	Stage 1 – BNWS up to 10 Early *	Stage 2 – BNWS up to 10 and number words before Early*/**	Stage 3 – BNWS up to 20 and number words before Early **/***	Stage 4 – BNWS up to 100 and number words before First *	Stage 5 – BNWS up to 1000 and number words after First **/***	Stage 6 – BNWS up to 1 000 000 and number words after Second */**/***

Grouping and place value/Subitising/ Estimating

Can use dice patterns and pattern and problem cards). Also think about the perceptual skills a pupil may demonstrate. *(Please make sure you assess estimation, as this can highlight difficulties with number sense).*

Grouping and Place Value							
Stage 0 – Emergent Early*	Stage 1 – Subitising small collections Early **	Stage 2 – Patterns of numbers up to 10/Ones as a Counting Unit Early ***	Stage 3 – Tens as a Counting Unit up to 100 First*	Stage 4 – Tens and hundreds as Counting Units up to 1000 First**/***	Stage 5 – Tens and hundreds in whole numbers/ Knows 10 tenths in a whole Second *	Stage 6 – Tenths in Decimals/Can order decimals with two decimal places Second **	Stage 7 – Tenths and hundredths in Decimals/Round to nearest tenth Second ***

Numeral Identification (Reading and Writing Numbers)

Can use with number id cards.

Numeral Identification							
Stage 0 – Emergent Numeral Identification Early*	Stage 1 – Can identify numerals to 10 Early **	Stage 2 – Can identify numerals to 20 Early***	Stage 3 – Can identify numerals to 100 First*	Stage 4 – Can identify numerals to 1000 First **/***	Stage 5 – Can identify numerals to 10 000 Second *	Stage 6 – Can identify numerals to 100 000 Second **	Stage 7 – Can identify numerals to 1 000 000 Second ***

Basic Facts

Problem cards may be useful in the assessment.

Basic Facts								
Stage 0 – Emergent Early*	Stage 1 – Finger Patterns to Five Early **	Stage 2 – Addition & Subtraction Facts to Five/ Doubles and Halves to Ten Early ***	Stage 3 – Addition & Subtraction Facts to Ten First*	Stage 4 – Addition Facts with Tens and Doubles/ Halves to Twenty First**	Stage 5 – Addition Facts to Twenty and Multiplication/Division Facts for 2, 5 and 10 First***	Stage 6 – Addition and Subtraction facts within 20 and multiplication Facts Second *	Stage 7 – Division Facts Second **	Stage 8 – Common Factors and Multiplies Second ***

Write a short description of the pupil's observed ability from assessment and in class in following sections.

Memorising and Mental Calculations

Has the pupil memorised some basic number facts e.g. 5+5=10, or 3x3=9?
Can they apply the basic facts they have memorised when solving mental calculations? (or do they use concrete materials, pen and paper, etc.?)

Mathematical Language And Symbols

Can the pupil use correct terminology in regards to numeracy and mathematics? E.g. plus/add/more than; minus/ take away/ less than; multiply/ times by; division/ into etc.?
Can the pupil correctly identify symbols such as +, -, x, = etc.? Once they know the symbol can they correctly solve the problem?

Strategy Assessments

Tick and date the stage of pupils demonstrated ability during the diagnostic interview assessment.

Addition and Subtraction

These problem cards may be useful. Guidance from the Highland Numeracy blog regarding Addition and Subtraction assessments can be found here.

Addition and Subtraction - Stage of Thinking							
Stage 0 – Emergent Early*	Stage 1 – Counts one- to – one but can't join sets Early **	Stage 2 – Counting from 1 with materials Early***	Stage 3 – Imaging Early ***	Stage 4 – Counting on/ back First *	Stage 5 – Early Additive First **/***	Stage 6 - Advanced Additive Second */**	Stage 7 – Advanced Multiplicative Second***

Multiplication and Division

Problem cards to be used with the assessments. Guidance regarding multiplication and division assessment can be found here.

Multiplication and division – Stage of Thinking					
Stage 0 –1 Emergent Early*/**	Stage 2 – 3 Counts in ones to solve problems Early***	Stage 4 – Skip-counting First *	Stage 5 – Early Additive – forms factors from known facts or repeated addition First **/***	Stage 6 - Advanced Additive – Derives from known multiplication facts Second */**	Stage 7 – Advanced Multiplicative – Uses at least two strategies Second***

Fractions

Assessment of fractional numbers. Problem cards may help with the assessment.

Fractions: Fractional Numbers - Stage of Thinking					
Stage 2-3 – Unit Fractions not recognised Early ***	Stage 4 – Unit fractions recognised First*	Stage 5 – Unit fractions ordered First**/ ***	Stage 6 – Co-ordinated numerals and denominators Second */**	Stage 7 – Equivalent fractions recognised Second***	Stage 8 – Mixed fractions ordered Second***

Assessment of fractions, proportions and ratios. Problem cards may help with the assessment.

Fractions: Fractions, Proportions, and Ratios - Stage of Thinking						
Stage 0 - 1 – Unequal sharing Early*/ **	Stage 2 Equal sharing with materials Early ***	Stage 3 Equal sharing by imaging Early***	Stage 4 – Skip counting First*	Stage 5 – Early additive – part whole Uses trial and improvement to solve problems with addition facts First**/ ***	Stage 6 – Advanced additive – part whole Uses a combination of addition facts and multiplication Second */**	Stage 7 – Advanced Multiplicative - Early Proportional Uses division and multiplication Second***

Write a short description of the pupil's observed ability from assessment and in class in following sections.

Word Problems

Can the pupil convert a word question into a numerical problem? Once they have done so can they solve the problem? Do they evaluate their answer?

Written work

Can the pupil correctly write numerals? Can the pupil use the correct symbols? Does a pupil lay out the problem or answer in a correct format (e.g. line up the columns for a chimney sum)?

Summary

Include strengths, areas of difficulty and relevant contextual information.

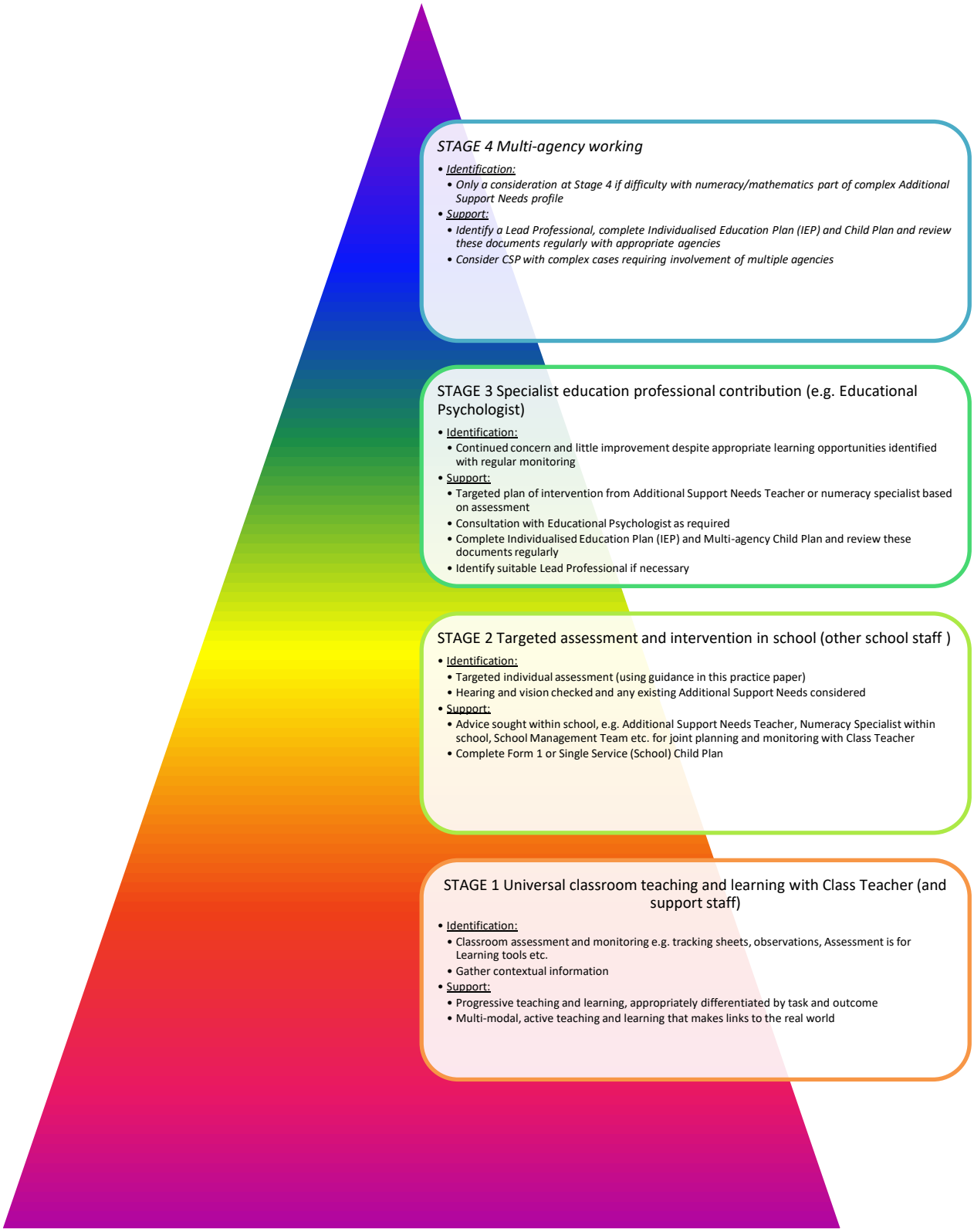
Recommendations for Numeracy/Maths Teaching

Based on the results of these assessments, the following recommendations are being made:
Refer to Highland Council Psychological Service's 'Assessment And Strategies For Teaching Numeracy' Practice Paper for suggested strategies.

Signed:	**Date:**

Appendix B: A Staged Approach to Identifying and Responding to Additional Support Needs

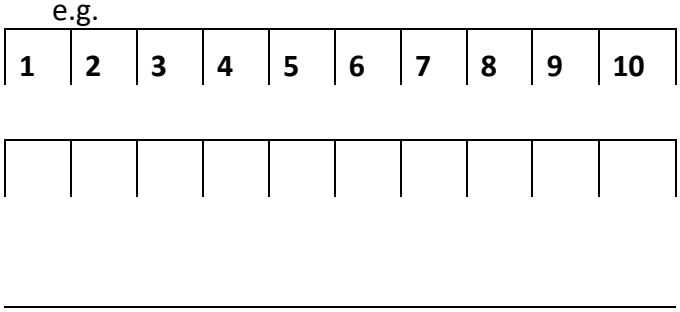
This diagram has been created based on the Service Delivery diagrams from Highland's Child 4 and Children in the Highlands Information Point (CHIP+) documents.



Appendix C: Tables of Difficulties Associated with Learning Numeracy

Number Sense and Applications of Arithmetic Procedures	
Features	**Possible Strategies and Resources (more suggestions on Highland Numeracy Blog)**
• Frequent use of immature strategies, i.e. strategies frequently used by younger children which have impact on pace and on memory • Frequent errors applying arithmetic procedures (e.g. addition, subtraction, multiplication, division) • Not confident giving answers to maths problems, even when correct • Lack of ability to estimate whether answer is roughly correct • Difficulties sequencing numbers and/or procedures appropriately • Difficulties counting, especially when numbers become larger (e.g. bigger than 30) • Poor one-to-one correspondence • Reliance on concrete materials for counting and procedures, lack of ability to understand number as abstract concept • Inability to subitise (immediately recognise quantity of small amount of objects, usually <5)	• Safe, reassuring learning environment with praise for effort and recognition of success • Concrete resources and visual diagrams to explain quantities and procedures, making links between the two. • Finger counting activities with hands in view and not in view (e.g. counting on fingers under table or behind head, 'bunny ears') to encourage visualisation • Reinforcement of number patterns , e.g. dice patterns, Numicon, card patterns • Use number lines and number squares in counting and 'skip counting' (counting in 2s, 3s etc.) se the pupil can see patterns • Move from concrete representation of number to pictures then finally to abstract by 'screening' (covering pictures or counters to encourage visualisation) then moving to written maths • Frequent opportunities to practise procedures in short bursts (try to avoid rote learning). Use of flash cards/repetition would support over-learning and familiarisation. • Teach strategies that would reduce cognitive load e.g. Use of doubles, near doubles etc. in times tables may be helpful • Active, multi-modal teaching and learning (visual, auditory and kinaesthetic) to encourage visualisation • Break down procedures using backwards and forwards chaining to teach procedures in progressive steps • For times tables, build up concept of what multiplication is using physical objects to group/share, skip counting (forwards and backwards), then pictures of quantities (visuable then screened) prior to learning 'tables' as facts

Remembering Facts About Numeracy and Mathematics	
Features	**Possible Strategies and Resources (more suggestions on Highland Numeracy Blog)**
• Reliance on counting methods rather than memory retrieval methods when working out arithmetic problems • Difficulties retaining and recalling mathematical facts, e.g. simple arithmetic answers, times tables, what procedures symbols mean (+, -, = etc.) • When facts are recalled, they are often wrong • Recall errors are frequently associated with the numbers in the maths problem e.g. answering 4 for 2+3=? because 4 is next in sequence after 2 and 3 • Inconsistent rates of recall; either slower because the pupil is counting or quicker because the pupil is guessing • Pupil can say right answer verbally but makes mistakes when asked to write it down (forgets) • Give incorrect answer in mental maths (guesses) but can recognise and write correction to answer when talking about answer (speaking slows down thing) • Makes mistakes or cannot recall procedures when asked to work independently after explanation despite showing understanding during teaching	• Learners need to be able to understand basic facts, even if these have not been rote learnt so reinforce conceptual understanding • Using an abacus specifically grouped by colour; e.g. five beads red, five beads blue and lots of strings (i.e. Rekenrek or Slavonic Abacus) is better for times tables than the ones with lots of different colour • Create a visual for basic facts e.g. use of five frames for facts to five, use of five-wise tens frames for five and facts (e.g. 5 + 2 = 7, 5 + 4 = 9 etc.), use of pair-wise tens frames from doubles and near doubles. Also use other equipment e.g. Rekenrek for same purpose but in different context to support application of understanding • Encourage pupils to record their working and thinking as they go along. This will allow them to track their work and answers • Support learners to work through a process of being able to see materials, then screen (cover up) materials to encourage visualisation of them, then remove materials altogether • Gradually build in links to written forms of these 'facts' but do so gradually after developing initial confidence with materials. • With inconsistent recall rates or errors, use real life contexts and applications (word problems, problem solving etc.) to increase relevance and encourage the pupil to make real life connections • Use questioning to help the pupil reason their own answer (and how reasonable it is) rather than telling and showing them what's wrong (links with meta-cognition and thinking strategies) • Specific activities and strategies that focus on memory may be of benefit as it may be a general memory difficulty that is causing the issues rather than an issue with maths/numeracy specifically

Visuo-Spatial	
Features	**Possible Strategies and Resources (more suggestions on Highland Numeracy Blog)**
• Difficulties in spatially representing a number line • Lack of a mental number line • Lack of innate tendency (in Western cultures) to count from left to right (even the left-handed are likely to do this) • Confusion between left and right causing difficulties with place value and counting forwards and backwards on a number line • Difficulties with shape, angles and analogue time • Difficulties with organising written work (e.g. putting answers with correct question numbers, writing vertical representation of addition or 'chimney sums' correctly etc.), • Difficulties with shape and geometry work, e.g. relating two-dimensional drawings to three-dimensional shapes. • Cannot see that the same shape or pattern once it has been moved, rotated or has a new orientation • Difficulties drawing and reading graphs • Difficulties in counting concrete objects and counting each item only once (this could also be due to motor planning/ co-ordination difficulties or due to difficulties other than maths e.g. visual impairment • Conservation of quantity, e.g. understanding that two differently shaped glasses may hold the same quantity of a liquid if one has a higher surface level	• Work on forward and backward number word sequences using numeral rolls and numeral tracks visible then progressing to screened. • Work on sequencing and ordering numbers in various ways depending on ability and depending on difficulty experiencing e.g. in 1s, in 2s, in 5s, in 10s on the decade and off the decade. (see page 27 of numeracy progression or use hyperlink.) • Order numbers which are not regularly spaced to get gauge of concept of size of different numbers and whether a number is greater or fewer in value than another • Work on a range of number lines e.g. those that are scaled and numbered (initially in 1s) but then working up to those that are scaled but not necessarily numbered and then onto empty number lines. Before being expected to use empty number lines to solve e.g. addition/subtraction problems, look at ordering numbers on scaled and empty number lines progressively. e.g.  • Left to right difficulty may also be helped by using numeral rolls and work on number lines as described above • Presenting numbers vertically may be more intuitive for pupils as the greater quantities are higher • Using bigger dots/ bigger resources can help pupils with motor or co-ordination or visual difficulties.

Maths Anxiety	
Features	**Possible Strategies and Resources (more suggestions on Highland Numeracy Blog)**
• a negative emotional response to situations involving number and mathematics • a range of emotional states that vary in intensity from mild apprehension or dislike to acute feelings of genuine fear or dread • avoidance of situations involving numbers of maths • quickly becoming anxious during numeracy tasks • anxious symptoms: breathing quickly, heart racing, tense, avoidant, challenging behaviour, fidgeting, feeling sick, self-soothing behaviours • either calculating very slowly and relying on counting or immature methods or rushing answers and making errors • memory, attention and concentration difficulties	• early identification and intervention will reduce impact • have an emotionally safe learning environment where it is safe, even good, to make mistakes • reinforcement of basic skills with visual and concrete prompts • encourage use of supports and note taking to support memory and focus • reduce exposure to negative attitudes about maths from others (teacher or parent may need to manage own anxieties) • discuss symptoms of anxiety so they can be recognised and addressed before they become overwhelming • cognitive behavioural approaches • psycho-education (learning about anxiety, how it affects the body and thinking, and how it can be controlled) • reframe language about maths to become more hopeful, forward looking and positive, e.g. “I will have to practice this skill a bit more before I can use it easily” • try a relaxation exercise before and after numeracy tasks • get the learner to write down or draw worries and concerns before starting a numeracy activity so they process their concerns before they process the numeracy task • read maths-related fiction books that are age and stage appropriate (e.g. suitable for younger readers or older readers) to explore feelings by discussing fictional characters • reinforce familiar skills and knowledge and only slightly stretch to learn new skills and knowledge for short periods • Keep tasks short to prevent fatigue and increase likelihood of success • Try to coach the learner through a numeracy task by asking questions and discussing feelings, rather than giving advice, clues or part of the answer

Appendix D: Persistent Difficulties with Numeracy and Additional Support Needs

As stated previously, learners can present with challenges in numeracy alone, or a combination of numeracy and other learning difficulties. **Not all those who present with numeracy challenges should be considered as having a specific learning difficulty with numeracy and/or maths.**

Many of the characteristics associated with some identified Additional Support Needs are associated with a difficulty with acquiring numeracy and maths knowledge and skills(BDA, 2015; Butterworth & Yeo, 2004; Emerson & Babtie, 2013). These include developmental and neurodevelopmental conditions such as:

- Attention Deficit Hyperactivity Disorder (ADHD)
- Sensory processing and integration disorder (sometimes, but not always, associated with Autism Spectrum Condition/Disorder)
- Specific language difficulties
- Persistent difficulties with literacy (Dyslexia; Specific Learning Difficulties in reading and/or writing)
- Developmental Coordination Disorder

There have also been research studies showing a correlation between delayed or atypical development of mathematical abilities and some genetic disorders affecting brain development. These include:

- Velocardiofacial syndrome (e.g. Simon, Bearden, McDonald Mc-Ginn & Zackai, cited in Fias & Fischer, 2005)
- Williams Syndrome (Paterson, Girelli, Butterworth & Karmiloff-Smith, cited in Butterworth, 2005)
- Turner Syndrome (Mazzocco & McCloskey, 2005)
- Fragile X Syndrome (Mazzocco & McCloskey, 2005)

Other Additional Support Needs related to social, emotional and behavioural difficulties and mental health would also be disruptive to learning of numeracy and maths; they could disrupt educational opportunities to acquire the necessary foundation knowledge and skills for numeracy and maths. Maths anxiety in particular can have a huge impact on maths learning.

The impact of difficulties in various cognitive abilities are described in the table below (information adapted from Emerson & Babtie, 2013 with additional information from other references):

Attention: Difficulties with attention can affect a learner's ability to concentrate and take in information. The ability to select the relevant information from instructions and have a logical approach when using procedures is needed in order to complete numeracy tasks. A learner with attention difficulties may need to be supported to do this with prompts, questions and screens for irrelevant information. Maintaining focus may also take more effort so may be more tiring.

Literacy (reading and writing): If learners have support needs in learning maths vocabulary, reading and writing then they may find numeracy work more challenging to understand and/or have difficulty with written numeracy work. Those with reading difficulties may also read larger numerals inaccurately or read word problems incorrectly.

Language, speech and poor auditory discrimination: Numeracy and maths have very specific language so if a learner takes longer to learn how to say the words or has trouble distinguishing sounds then it could take longer to acquire the vocabulary required and apply it correctly.

Memory:
Long-term memory is required in numeracy to remember key facts and procedures in order to carry out calculations. It may take longer to code these facts and the learners may require multi-sensory teaching so it sticks in the memory. If the learner has to rely on (immature) counting methods to work out arithmetic and does not learn simple sums and tables 'off by heart' then it takes longer and requires a greater cognitive load (mental activity). Learners have to rely more on their working memory to work out calculations.

Short-term memory is required to recall the instructions given for a task, so a learner may need a visual reminder or to make their own notes. They may also start a task then forget the question or key information from the instructions before reaching an answer.

Working memory is the ability to hold information in the mind and manipulate it, or work things out in the 'mind's eye'. The difficulties are apparent in mental maths tasks, especially those involving multiple steps and procedures. These learners may not pick suitable strategies or follow procedures correctly in their calculations by forgetting or mixing up steps in the procedures. They may need to write down working to keep track.

Coordination, motor skills and sensory integration: Difficulties coordinating fine and gross motor movements will make it more challenging to locate and look at the correct information on a page or diagram, difficulties writing and organising written work neatly and also difficulties eye tracking along text or number lines. These support needs could lead to errors being made in poorly presented written work and teaching materials.

If learners have poor body awareness they may also find counting more difficult because counting using their fingers would be harder, require more effort and may not help their understanding in the same way (due to not knowing where your fingers or hands are in space). This could lead to the learner trying to use other methods that are possibly too advanced or inappropriate and could lead to errors, such as calculating mentally, relying on memory or guessing. They may also find one-to-one correspondence in counting more difficult.

Visuo-spatial awareness including left-right orientation: Spatial awareness in required for using numbers lines as, generally, counting forwards (so the numbers increase) goes left to right and counting backwards (so the number decrease) goes right to left. However, in place value, the numeral representing the 'tens' goes on the left (e.g. 54) and the numeral representing the ones (or units) is on the right (e.g. 54). If these are mixed up, the calculations will result in the wrong answers. Learners with visuo-spatial difficulties may also find visually presented information difficult to interpret and have trouble lining up written work appropriately in columns when required (e.g. column addition and subtraction, also known as 'chimney sums', long multiplication and long division etc.)

Research has suggested that individuals who are blind from an early age often have a better sense of number and better mathematical ability than many sighted people (Kwon, 2016). This was thought to be because these blind people have developed compensatory mechanisms that seem to foster a better understanding of number and mathematical concepts. One theory is a blind person cannot rely on visual cues or written materials so may develop a stronger working memory (which helps maths ability) and another theory is blind children spend a lot of time touching and manipulating objects to understand numerical information as they grow and develop, so rather than mapping numbers on visual space the blind map numbers onto their bodies (Crollen, Dormal, Seron, Lepore & Collignon, 2013; Kwon, 2016). By having a different mental conception of space that does not rely sight, blind people could develop a better understanding of three-dimensional space and abstract mathematical concepts (Kwon, 2016).

Difficulty with abstract reasoning: Mathematical concepts can be quite abstract, for example, the concept of 'three' is represented in multiple ways that are different yet all related: the numeral (3); the spoken word ("three") the written word (three); a small set of similar objects; a small set of different objects; a small set of ideas etc. If abstract thinking is difficult for the learner, they may rely more heavily on concrete supports for counting and calculating such as small objects like counters and number lines or squares. These learners may not grasp number concepts as easily or quickly as their peers and will need a lot of reinforcement.

Processing speed: Learners who process information more slowly will need extra time for their calculating. They may also need to write down their working or memory prompts so they do not forget the instructions, miss out steps of procedures or key information form the instructions. They may also need spoken instructions to be repeated without rephrasing so they can process the information fully without becoming confused by a reworded statement.

Sequencing: Counting and other procedures in arithmetic require a set sequence of actions that does not change and those with sequencing difficulties may miss out or mix up the sequences of numbers, days of the week, calendar months, times of day, or steps in a procedure. Some procedures have commutativity (the numbers can be in any order, e.g. 2+3=3+2 or 2x3=3x2) and some do not (3-2=1 whereas 2-3=-1) and so it is essential to know when the order is important in order to get the right answer. Those with sequencing difficulties may also struggle how to order their written work and logical reasoning.

Planning and Organisation: Numeracy and maths activities demand a structured approach or the likelihood or careless mistakes increases. As with other difficulties, this would have an impact on multi-step procedures, problem solving, logical reasoning and written work. These learners require support to think and work in an ordered way so they do not miss out or mix up parts of the numeracy activity. If planning and organising is a difficulty, the learner may need help to layout their work correctly so they do not end up with errors, they may also need help to figure out how to start to answer a numeracy question.

Appendix E: Scottish Qualifications Authority (SQA) and Assessment Arrangements

Assessment Arrangements can be provided for candidates who have identified specific and significant difficulties in number. This is sometimes referred to as dyscalculia, however, it is important to note that the SQA has a needs-led model.

This means that there is **no preferred term which has to be used by the school (or exam centre) to describe the candidate's identified difficulty** as long as the guidance for requesting Assessment Arrangements is followed and supported with sufficient evidence (see guidance http://www.sqa.org.uk/sqa/14976.html).

Candidates with an identified difficulty with number requiring an Assessment Arrangement, including the use of a numerical aid, may persistently display the following in their learning (this is not an exhaustive list):

- Difficulty in understanding numbers,
- Difficulty with place value (hundreds, tens, units)
- Confusion with mathematical symbols
- Losing track when following procedures, for example long multiplication
- Significant difficulty in learning times tables
- Directional confusion, e.g. Instead of going right to left with addition, subtraction and multiplication, will work the other way
- Difficulties in sequencing
- Difficulties with time and/or measurement

As long as a school has robust evidence that a candidate is placed at a substantial disadvantage because of their particular difficulties, it would be acceptable to request the use of a multiplication square, number line or basic arithmetic calculator in those SQA assessments where such support is currently restricted. For National Qualifications, these are:

- the first part of the assessment in the Added Value Unit at National 4 and in the Course assessment at National 5 and Higher in Mathematics
- the first part of the assessment in the Added Value Unit at National 4 and in the Course assessment at National 5 in Lifeskills Mathematics

Note* *In all other Unit and Course assessments in Mathematics and Lifeskills Mathematics, any candidate can use a number/multiplication square, number line, basic arithmetic calculator or equivalent technologies.*

Candidates for whom such requests may be necessary may have severe difficulties with basic arithmetic and recall of number facts. This would mean that they would be at a disadvantage in an mathematics assessment that required a competency in basic mental arithmetic as part of multi-step calculations and responses.

All requests for Assessment Arrangements in SQA assessments and examinations would have to be supported with evidence that described: the nature and degree of the candidate's difficulty; the ongoing support arrangements provided to allow the candidate to access the course/class work; how the support was appropriate and how the candidate would be placed at a disadvantage without the support arrangement.

Guidance and examples of good practice and evidence requirements to support a request for Assessment Arrangements can be found on the SQA website (http://www.sqa.org.uk/sqa/14976.html).

There is also information regarding SQA's Quality Assurance requirements which must be adhered to. The verification meeting would provide an opportunity to discuss and record all of the information gathered about a candidate's assessment needs and the impact of the candidate's identified disability or difficulty on accessing teaching and assessment.

This guidance was written with the collaboration of the Scottish Qualifications Authority through correspondence by email.

Appendix F: Suggested Support Materials for Classroom Use: Annotated Bibliography

This is a list of resources that could be used to support assessment and intervention for difficulties with numeracy. This is not an exhaustive list as there will be other resources available currently and in the near future as this is a developing area of research. These resources are descriptions of some resources that are available and not necessarily endorsements.

Books and Other Materials

Butterworth B. (2003). ***The Dyscalculia Screener***, GL Assessment This resource is accessed using a CD or online from the GL assessment website. Part of the Dyscalculia Screener manual that gives some descriptive detail about dyscalculia is also on the GL Assessment website as well as a demo of the screener. It is a computer-based screener and should not be used in isolation as an assessment of persistent difficulties with numeracy, but combined with other assessment information can form important additional information. A disadvantage is that it either requires an annual licence or if used online the individual assessment administrations cost £5.70 each (+VAT, price correct 23/08/16) and the minimum number that can be bought at a time is ten. This resource, if used, is probably best as an area resource for Support for Learning teachers, or jointly funded by all schools within an ASG.
Butterworth B. & Yeo D. (2004). ***Dyscalculia Guidance,*** GL Assessment This could form the basis of an intervention strategy following assessment. It includes activities and games for early stages leading onto "harder" addition and subtraction, working with larger numbers, times tables and fractions.
Chinn, S. (2012). ***Maths Learning Difficulties, Dyslexia and Dyscalculia***, British Dyslexia Association (BDA) Written by an experienced and respected teacher, educator and researcher this book is very accessible and has practical advice and strategies for teaching numeracy and maths to those with Additional Support Needs. It is relatively inexpensive and available from the British Dyslexia Association shop.
Emerson J. & Babtie, P., (2012). ***The Dyscalculia Assessment,*** Bloomsbury This resource can be used as an alternative to the Highland Numeracy Progression Diagnostic Assessments from the Highland Numeracy Blog to form the basis of a comprehensive numeracy assessment for teachers. Do not be put off by the amount of information contained in it as the Highland Numeracy Progression will inform which parts of the assessment are needed for an individual learner. The Appendices includes a sample report writing up the assessment, a summary Maths Profile and a questionnaire for teachers and parents. It also contains a list of useful websites and resources
Henderson A. (2012) ***Dyslexia, Dyscalculia and Mathematics***, Routledge, Now in its second edition, this book continues to have good ideas which can be used for assessment and intervention.

Maths Recovery

Wright R. J., Martland, J., Stafford, A. K. (2006a). ***Early Numeracy: Assessment for Teaching and Intervention (Second Edition) (the blue book)***, Paul Chapman Educational Publishing

Wright, R. J., Stanger, G., Stafford, A. K. and Martland J. (2006b). ***Teaching Number: Advancing Children's Skills and Strategies (Second Edition) (the green book)***, Paul Chapman Educational Publishing

Wright, R. J., Stanger, G., Stafford, A. K. and Martland J. (2014). ***Teaching Number in the Classroom with 4-8yr olds (Second Edition) (the purple book),*** Paul Chapman Educational Publishing

Wright, R. J., Ellemor-Collins, D., Tabor P. D. (2011). ***Developing Number Knowledge: Assessment, Teaching and Intervention with 7-11 year olds (the red book)***, Sage Publications Ltd

Wright R. J., Hackenberg A. J., Norton, A. (2016) ***Developing Fractions Knowledge (the orange book)***, Sage Publications Ltd

http://www.mathsrecovery.org.uk/

Maths Recovery is an evidence-based intensive assessment and intervention process for targeting pupils who are having difficulties with numeracy and maths. A number of teachers have now been trained in the Maths Recovery approach within Highland and their advice can be sought by contacting the Quality Improvement Officer with responsibility for Numeracy (currently Lesley Taylor) or the Numeracy Development Officer (currently Kirsten MacKay).

The books describe the Maths Recovery Assessments (*Early Numeracy: Assessment for Teaching and Intervention* or 'the blue book'), the intensive intervention with individuals and small groups (*Teaching Number: Advancing Children's Skills and Strategies* or 'the green book'), and classroom strategies for teaching maths (the remaining books).

Websites

https://education.gov.scot/improvement/Pages/CfE-delivery-plan.aspx

This link is for "Delivering excellence and equity in Scottish education: A delivery plan" from the National Improvement Hub. There are several guidance documents about delivering Curriculum for Excellence including the benchmarks for Numeracy and Mathematics.

http://www.educationscotland.gov.uk/resources/n/nnmpf/

This link is for the National Numeracy and Mathematics Progression Framework 2016. This resource was produced to help develop understanding of the progression through the Experiences and Outcomes. It shows the progression milestones in numeracy and mathematics.

https://highlandnumeracyblog.wordpress.com/

https://highlandnumeracyblog.wordpress.com/assessments-2/

These are the links for the Highland Numeracy Blog and the Highland Numeracy Progression Diagnostic Assessment sheets discussed in this practice paper. There is also a very useful glossary of mathematical terms. There is training available on how to use the Highland Numeracy Progression and the Highland Numeracy Progression Diagnostic Assessment sheets is offered regularly in Highland, please contact the Numeracy Development Officer(currently Kirsten MacKay) for details.

https://highlandnumeracyblog.files.wordpress.com/2015/01/hnp.docx

This is the link to the Highland Numeracy Progression, which is a numeracy planning and teaching resource based on Curriculum for Excellence, New Zealand Maths and Maths Recovery.

https://nzmaths.co.nz/numeracy-projects

This is the website for the New Zealand Maths which has free downloadable professional development resources and teaching resources that are used in many primary schools in Highland.

https://highlandnumeracyblog.files.wordpress.com/2015/01/counting-and-part-whole-strategies-hnp_nz_mr-comparison1.docx

This document can be found on the Highland Numeracy Blog and compares the stages of the Highland Numeracy Progression with New Zealand Maths. Please contact the Numeracy Development Officer(currently Kirsten MacKay) for more details.

http://www.stevechinn.co.uk/

This is Steve Chinn's website that has some information **maths learning difficulties and dyscalculia** based on his books and research. There are also links to video tutorials and an interesting Maths Anxiety Quiz for adults.

http://www.mathematicalbrain.com/

This site has some detail about Brian Butterworth (who wrote the Dyscalculia Screener and other texts about Developmemntal Dyscalculia) and his book *The Mathematical Brain.*

http://www.bdadyslexia.org.uk/dyslexic/dyscalculia

This link is for a page on the website for the British Dyslexia Association (BDA), which has good up-to-date information about the definition and prevalence. It is mostly based on the English Education system. On this page there are links to a general information sheet by the BDA as well.

http://www.dyslexiaaction.org.uk/dyscalculia

This is the Co-occurring Difficulties page on the website for Dyslexia Action, a national dyslexia charity. There is information on Dyscalculia on the page if you scroll down.

http://www.dyscalculia.me.uk/

This link is the website for the Dyscalculia Centre which has information for parents and teachers. There is also a dyscalculia test available on this website that is expensive (£59.95, or £49.95 for three, price correct 23/08/16). Highland Council's policy on private reports is that they are taken into account however schools and allied services do not necessarily have to agree with their conclusions. A report such as this **would not** be necessary for any Assessment Arrangements or to access support in schools.

http://www.dyscalculia.me.uk/SEN64%20dyscalculia.pdf

This is a link to an article from SEN Magazine Issue 64 dated July/August 2013 discussing Dyscalculia by Tony Attwood, founder of the Dyscalculia Centre. You will have to scroll down for the article.

Appendix G: Acknowledgements

This paper was mostly written by Stephanie Bennett and Isabel Martland from Highland Council Psychological Service but its creation was a collaborative effort with other education professionals within and out with The Highland Council. The authors would also like to thank their school partners with whom numeracy assessments and interventions were discussed. These consultations in practice helped inform the guidance included in this practice paper. The Highland Council Psychological Service would like to specifically thank the following people who contributed to this practice paper with content or suggestions:

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Validated Self Evaluation

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- Susan Bryce, Additional Support Needs Teacher, Dornach Primary
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- Jill Kerr, Principal Teacher of Additional Support Needs, Millburn Academy
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- Miriam MacDonald, Head Teacher, Crown Primary School
- Kirsten MacKay, Numeracy Development Officer
- Oona Oakland, Additional Support Needs Officer and Peripatetic Additional Support Needs Teacher, Skye and Lochalsh
- Garry Simpson, Principal Teacher of Mathematics, Kinlochleven High School
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- Alison Sangster, Educational Psychologist
- Kaylee MacPhee, Educational Psychologist (Probationer)
- Victoria Shephard, Educational Psychologist (Probationer)

It should also be noted that this practice paper refers to the Highland Numeracy Progression which was created by a local authority numeracy development group within The Highland Council. The associated diagnostic assessment sheets were written by Kirsten MacKay, Numeracy Development Officer for Highland Council. Both numeracy resources were based on research and writing by Dr. Robert Wright and various co-authors (primarily the Stages of Early Arithmetical Learning or SEAL), Maths Recovery and New Zealand Maths.